RULES OF INFERENCE

1. Modus ponens (MP)
$p \supset q$
p
$\therefore q$

2. Modus tollens (MT)
$p \supset q$
$\sim q$
$\therefore \sim p$

3. Hypothetical syllogism (HS)
$p \supset q$
$q \supset r$
$\therefore p \supset r$

4. Disjunctive syllogism (DS)
$p \vee q$
$\sim p$
$\therefore q$

5. Constructive dilemma (CD)
$(p \supset q) \cdot (r \supset s)$
$p \vee r$
$\therefore q \vee s$

6. Simplification (Simp)
$p \cdot q$
$\therefore p$

7. Conjunction (Conj)
p
q
$\therefore p \cdot q$

8. Addition (Add)
p
$\therefore p \vee q$

Axiom of replacement: Within the context of a proof, logically equivalent expressions may replace each other.

9. DeMorgan's rule (DM)
$\sim(p \cdot q) :: (\sim p \vee \sim q)$
$\sim(p \vee q) :: (\sim p \cdot \sim q)$

10. Community (Com)
$(p \vee q) :: (q \vee p)$
$(p \cdot q) :: (q \cdot p)$

11. Associativity (Assoc)
$[p \vee (q \vee r)] :: [(p \vee q) \vee r]$
$[p \cdot (q \cdot r)] :: [(p \cdot q) \cdot r]$

12. Distribution (Dist)
$[p \cdot (q \vee r)] :: [(p \cdot q) \vee (p \cdot r)]$
$[p \vee (q \cdot r)] :: [(p \vee q) \cdot (p \vee r)]$

13. Double negation (DN)
$p :: \sim\sim p$

14. Transposition (Trans)
$(p \supset q) :: (\sim q \supset \sim p)$

15. Material implication (Impl)
$(p \supset q) :: (\sim p \vee q)$

16. Material equivalence (Equiv)
$(p \equiv q) :: [(p \supset q) \cdot (q \supset p)]$
$(p \equiv q) :: [(p \cdot q) \vee (\sim p \cdot \sim q)]$

17. Exportation (Exp)
$[(p \cdot q) \supset r] :: [p \supset (q \supset r)]$

18. Tautology (Taut)
$p :: (p \vee p)$
$p :: (p \cdot p)$